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Light Composite Vector Bosons

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Abstract

In gauge theories with slowly-running coupling constants, it may be possible for four-fermion operators to be nearly marginal. Such operators can possess asymptotically weak couplings, and can plausibly give rise to light composite vector mesons.

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The so-called Standard Model of accessible-energy particle physics, in spite of its increasingly precise experimental confirmation, remains unsatisfactory for several theoretical reasons. One of these is the hierarchy problem: it is unnatural for the Higgs scalar to appear in the theory at all, since its mass is not protected by any symmetry. Another is the replication of flavors and associated mass and mixing-angle parameters: these may be technically natural, but are puzzling nonetheless.

These theoretical defects have prompted the development of many extensions to the Standard Model over the past two decades. Sensible extensions should possess a limit in which they reduce to the Standard Model, as this allows them to explain why the latter is such a successful effective theory at currently accessible energies. Such extensions can solve the hierarchy problem: both supersymmetry and technicolor [1] models (with or without grand unification) do. The former protects a fine-tuning, while the latter also explains the origin of a small scale (compared to the other known scale, the Planck scale). The flavor problem, on the other hand, is evidently quite hard: supersymmetric models do not even address it, while the literature is littered with the corpses of extensions to technicolor models that have succumbed either to the presence of excessive flavor-changing neutral currents or to the inadmissibility of a heavy top-quark, or to both.

Both of these approaches presume that the flavor physics underlying the Standard Model can be explained purely in terms of weak-coupling physics, possibly supplemented by a scaled-up version of QCD. This is not necessarily wrong, but does seem a bit presumptuous, especially in light of the growing evidence that one of the parameters (the top Yukawa coupling) may be large enough to produce non-trivial dynamics. It thus seems sensible to search for other extensions to the Standard Model in which strong-coupling dynamics plays a role.

One step in this direction was taken by Bardeen, Hill, and Lindner (BHL) [2], following earlier work [3]. They introduced a Nambu–Jona-Lasinio–type [4] four-fermion coupling of the top quark with a large dynamically-generated anomalous dimension; at lower energies, it becomes strong, generating an $SU(2)\times U(1)$-breaking condensate, along with a composite Higgs scalar. In the limit originally considered by these authors, their model was later shown to be completely equivalent to the Standard Model [5]. Far from being a defect, this is in fact a virtue, as explained above. Interesting extensions may be obtained not by taking the BHL 'cutoff' (in their language) very high, but rather in taking it as *low* as possible without violating known experimental constraints [6].

In this letter, I explore another possible avenue for generating extensions to the Standard Model: rather than rewriting the Yukawa interaction in terms of four-fermion couplings, and generating a composite scalar, I rewrite the spontaneously-broken gauge interactions in terms of four-fermion operators, generating a light, composite, vector field. This idea is in fact nearly as old as I am [7,8,9,10]. The sensible attempts have concluded that it is both necessary (for consistency [11]) and natural for the couplings of the light vector to approach the form of gauge couplings at low energies. For reasons I will review below, however, these attempts have confined their attention to the low-energy effective theory beneath the binding scale, and have not been able to explore the possible origins of such a theory at high energies. What is new below is the observation that gauge theories with slowly-running couplings — so-called walking gauge theories [12,13,14] — allow us to do so.

Let me begin by paraphrasing previous analyses of a theory of N massive fermions with an attractive vector interaction,

$$\mathcal{L} = \overline{\psi}(\partial\!\!\!/ + M)\psi + \frac{f}{\Lambda^2}\left(\overline{\psi}\gamma_\mu T^a \psi\right)^2 \ , \tag{1}$$

where T^a are the generators of $SU(N)$ in the fundamental representation, and where Λ is a large scale ($\gg M$) where the four-fermion operator was originally induced. We may imagine evolving the lagrangian down to smaller and smaller scales; when we cross M, we should integrate out the massive fermions, since they will no longer be present in the theory at lower energies. As the various authors of ref. [10] show, when we do this, we will generate the action for an $SU(N)$ gauge field, with mass $\Lambda/\sqrt{f}$, along with various higher dimension operators which will be small for $p \ll M$.

Of course, we will only get to such momenta if the mass of the vector is also much less than M; this in turn requires a large f,

$$f \gg \left(\frac{\Lambda}{M}\right)^2 \ . \tag{2}$$

Were this reasonable, the presence of a light vector would be understandable: the strong attractive interaction in the vector channel binds the fermions deeply, lowering the mass of the vector two-fermion state far below its naive threshold value of $2M$.

Unfortunately, from an effective field-theory point of view, equation (2) is anything but reasonable, and it is certainly not possible to interpret the theory between the scales M and Λ as having the usual sort of weak-coupling effective Lagrangian we are used to.

As we will see, the picture can change considerably if we take the fermions to transform under a non-Abelian gauge interaction with a ‘walking’ coupling of sufficiently large value. The basic point is that such a walking gauge interaction (WGI for short) can induce large anomalous dimensions for four-fermion operators, perhaps sufficiently large to make operators like the one in the Lagrangian (1) nearly marginal (in the language of the renormalization group). (A related point was noted by Bardeen, Leung, and Love [15] in the context of quenched planar QED.) This alone would be sufficient to tame equation (2) to something like $f \gg 1$, but it is not quite the end of the story. For nearly marginal operators, it is important to consider self-renormalization effects, and as we shall see, an attractive operator like the one in the Lagrangian (1) is in fact asymptotically weak, so that $f(\Lambda)$ can be perturbatively small, yet nonetheless ultimately give rise to a light vector state.

To understand this concretely, consider the following Lagrangian,

$$\mathcal{L} = \overline{\Psi} \not{D} \Psi + \frac{f}{\Lambda^2} \left(\overline{\Psi} T^a \gamma_\mu P_L \Psi \right)^2 + \cdots \tag{3}$$

with l flavors of Dirac fermions Ψ transforming as an N_w of the walking gauge theory $SU(N_w)$, where T^a are the generators of the fundamental representation of $SU(l)$, and where $P_L = (1 - \gamma_5)/2$ is a left-chiral projection operator. (The dots indicate other fermions which do not participate in the four-fermion interactions, but are required in order to make the coupling walk.) In an $SU(N)$ theory with n_f fundamental Dirac fermions, the theory has a running coupling with the two-loop beta function [16],

$$\beta(\alpha) = -\frac{1}{2\pi} \left(\frac{11}{3} N - \frac{2}{3} n_f \right) \alpha^2 - \frac{1}{8\pi^2} \left(\frac{34}{3} N^2 - \frac{10}{3} N n_f - 2 C_f n_f \right) \alpha^3 \, . \tag{4}$$

In a walking theory, the matter content is such that the leading coefficient is much smaller than its canonical value (say with $n_f = 0$), while the second coefficient is smaller yet in magnitude. As a result, the coupling runs very slowly in the weak-coupling regime. For a theory to walk at interesting values of the coupling constant, we should require[†] $\beta(\alpha)/\alpha \ll 1$ even at finite couplings of order the critical coupling for chiral-symmetry breaking. Typical walking theories have $\beta(\alpha)/\alpha \sim -0.1$ to -0.3 for near-critical α's [14] in contrast to three-flavor QCD, which has $\beta(\alpha)/\alpha \sim -1.6$ for such couplings. Actually, we may choose to define this critical coupling α_c in two ways: either as the coupling at which chiral-symmetry breaking is triggered, or else the coupling at which the anomalous dimension for the fermion self-energy becomes exactly -1. (Cohen and Georgi [17] argue that these two definitions are equivalent even beyond perturbation theory.) Within the resummed-rainbow approximation,

$$\alpha_c = \frac{\pi}{3 C_F(N_w)} \, , \tag{5}$$

where C_F is the quadratic Casimir of the fundamental representation.

What happens to the four-fermion operator in the Lagrangian (3) under the influence of the gauge interactions? For one thing, it will mix with other operators, so we should consider the full set of operators which mix under renormalization-group flow. At first order in perturbation theory

[†] The beta function in fact becomes renormalization-scheme dependent beyond two-loop order, so it is not clear what this statement means. Anomalous dimensions of physical quantities calculated to all orders in perturbation theory will not suffer from this problem.

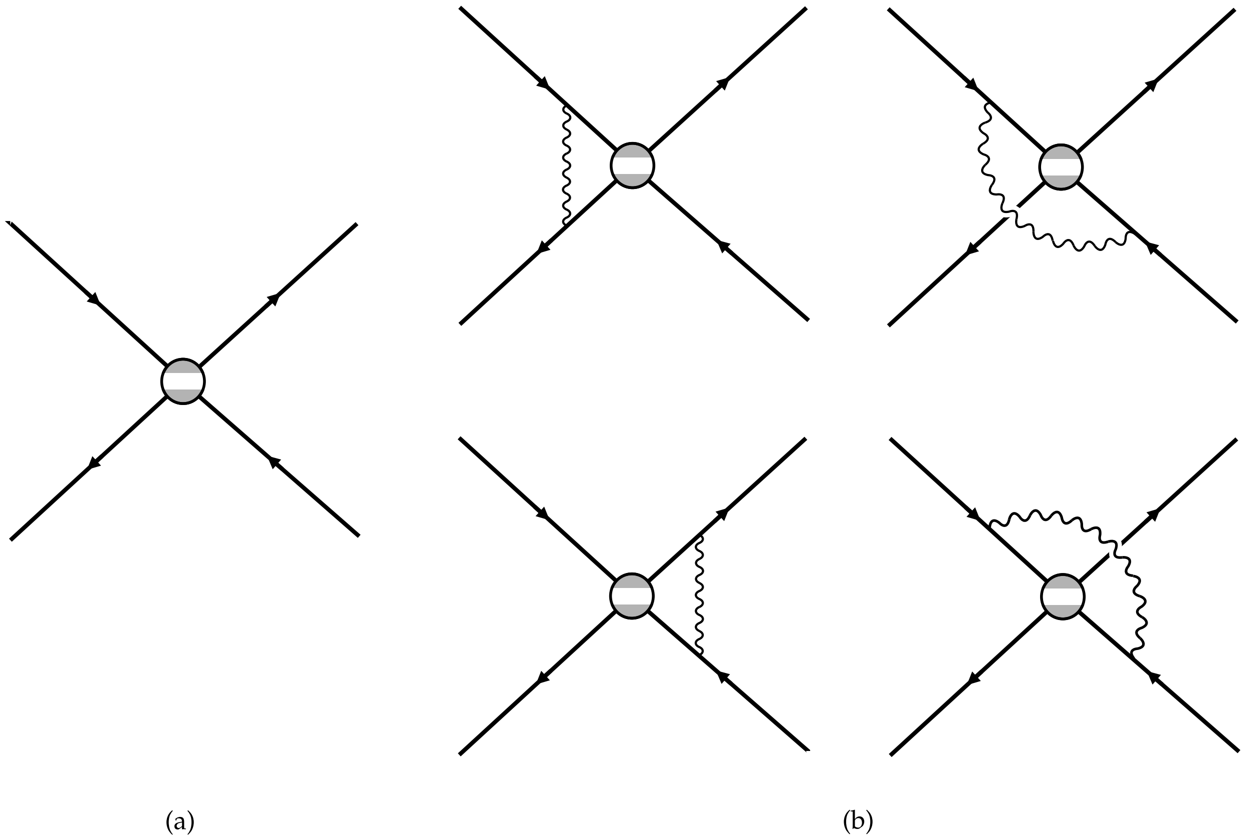


Figure 1 (a) The left-hand side of the Schwinger-Dyson equation for the four-fermion operator (b) The contributions to the right-hand side of the Schwinger-Dyson equation in the Landau-gauge rainbow approximation.

(and thus within the analog of the resummed-rainbow approximation), there are two such operators we must consider,

$$O_{\pm} = \frac{1}{2\Lambda^2}\left[\overline{\Psi}^x T^a \gamma_\mu P_L \Psi_x \, \overline{\Psi}^y T^a \gamma^\mu P_L \Psi_y \pm \overline{\Psi}^x T^a \gamma_\mu P_L \Psi_y \, \overline{\Psi}^y T^a \gamma^\mu P_L \Psi_x\right] \;, \tag{6}$$

where x, y denote WGI indices. The four-fermion operator in equation (3) is simply the sum of O_+ and O_-. (I have ignored the WGI penguin operator $\overline{\Psi}\tau^x_w\gamma_\mu P_L\Psi f_{xyz} D^y_\nu F^{z\mu\nu}$ for reasons which will be explained later, and possible non-perturbative effects such as instanton-induced mixing for simplicity.) The contributions of these operators to Green functions will have the form

$$(1 - \gamma_0 \ln(\Lambda/p) + \cdots)\; O \;, \tag{7}$$

which at finite constant coupling we might rewrite in the form

$$\left(\frac{p}{\Lambda}\right)^{\gamma_0} O + \cdots \tag{8}$$

In particular, if the perturbative anomalous dimension [18] γ_0 is negative, the operator will be enhanced at scale p compared to its naive strength of $(p/\Lambda)^2$. Can $|\gamma_0|$ be large enough to overcome this intrinsic suppression?

To go beyond weak coupling, we should in principle, solve the (matrix) Schwinger-Dyson equations generated (in Landau gauge) by the collection of diagrams of fig. 1. This is a rather difficult task. To get an idea of the qualitative behavior we might expect, however, we can proceed as follows. First, assume that the important mixing effects are those given by the perturbative anomalous dimension matrix; and that a multiplicatively-renormalized operator O_m scales simply according to the momentum flowing in the vector channel. Also, even though the fermion self-energy is enhanced compared to its value in an ordinary gauge theory, it is still small and hence can be neglected. This suggests the following simplification of the Schwinger-Dyson equation,

$$O_m(s) = -\frac{1}{\Omega_3} \int d^4k\; \gamma_0(\alpha(k)) \frac{O_m(k^2)}{(k-p_1)^2 (k-p_2)^2}\;, \tag{9}$$

where $\Omega_3 = 2\pi^2$ is the volume of the unit three-sphere. Even this reduced form is hard to understand analytically; but we may expect that at small momenta, the denominator will go as $\sim s^2$, where s is a typical invariant mass, while at high momenta the invariant mass will be irrelevant, and the denominator will go as $\sim k^4$. We may also work in the 'standing' approximation in which the coupling does not run at all. Following this reasoning, we arrive at the following caricature of the original equation,

$$O_m(s) = -\gamma_0 \int_0^{\sqrt{s}} k^3 dk\; \frac{O_m(k^2)}{s^2} + \int_{\sqrt{s}}^{\Lambda} \frac{dk}{k}\; O_m(k^2)\;. \tag{10}$$

If we now plug in an ansatz for the form of $O_m(s) \sim s^{\gamma/2}$, perform the integrals, and extract the pieces proportional to $s^{\gamma/2}$, we can derive the algebraic equation,

$$1 = \frac{4\gamma_0}{\gamma(4+\gamma)}\;. \tag{11}$$

For γ_0 small, this just yields the perturbative result, $\gamma = \gamma_0$ (we discard the other solution); in general, we find

$$\gamma = -2 + 2\sqrt{1+\gamma_0}\;. \tag{12}$$

As $\gamma_0 \to -1$, the anomalous dimension becomes large enough that the four-fermion operator, of naive dimension six, can become nearly marginal: an anomalous dimension of $-2+\delta$ implies an effective scaling dimension of $4+\delta$, i.e. that of a marginally irrelevant operator. Furthermore, the anomalous dimension is *larger* in magnitude than would be indicated by a linear extrapolation of perturbation theory, a feature that is also true of the self-energy anomalous dimension within the rainbow approximation. (We should actually take the argument of O_m in the first integral in equation (11) to be something like k^2+s; this does not change the qualitative statements made here, though the numerical values of the anomalous dimension γ become somewhat smaller.)

Unfortunately, we also hit our first obstacle here. If we rewrite the perturbative coefficient γ_0 in terms of α/α_c, we find

$$\gamma_0 = -\frac{r}{2}\left(\frac{\alpha}{\alpha_c}\right) , \tag{13}$$

where r is a group-theoretic coefficient. If we consider a more general fermion representation, with $\Psi_{1,2}$ transforming in representations $R_{1,2}$ of the WGI, and four-fermion operators

$$O = \frac{1}{\Lambda^2} M_{xyz} M_{\hat{x}\hat{y}z} \overline{\Psi}_1^x T^a \gamma_\mu P_L \Psi_1^{\hat{x}} \, \overline{\Psi}_2^y T^a \gamma_\mu P_L \Psi_2^{\hat{y}} , \tag{14}$$

where the matrices M are the Clebsch coefficients for $R_1 \otimes R_2 \to R_3$, and $x, y, z, \ldots$ represent sets of WGI indices, then

$$r = \frac{C_2(R_1) + C_2(R_2) - C_2(R_3)}{\max\left(C_2(R_1), C_2(R_2)\right)} . \tag{15}$$

($C_2(R)$ is the quadratic Casimir of representation R.) If $R_1 = \overline{R}_2$, and we take at $R_3 = 1$, then $r = 2$; but in this case (which corresponds to a mixed left-right operator in QCD), the relevant four-fermion operator presumably contributes in the appropriate color-singlet *scalar* channel (and also contributes to destabilizing the chirally-symmetric vacuum). For other cases, R_3 cannot be the identity, and $r < 2$. In particular, if we stick to fundamental complex representations in vector-like theories, then in fact we find $r \leq 1$ for $SU(N)$. More generally, the best one seems to be able to do with complex representations while maintaining asymptotic freedom is with third-rank antisymmetric tensor representations of $SU(7)$, for which $r = 3/2$. If we take the form of equation (12) seriously, the fact that $r < 2$ implies that the coupling will have grown large enough to trigger chiral-symmetry breaking before it has gotten large enough to make any LL four-fermion operator nearly marginal. Of course, in the full theory, the form of this equation should not be taken too seriously; and it may well be that the theory enhances the four-fermion anomalous dimensions more at strong coupling than the corresponding one for the fermion self-energy. In this case, a four-fermion operator could become nearly marginal at couplings smaller than that needed to trigger chiral-symmetry breaking. Even without hoping for such a miracle, however, there are other avenues of escape from this impasse. I will mention two of them below.

For the moment, let us simply ignore this issue, and imagine that we can find a theory with $r \simeq 2$, or something equivalent to it. Take the coupling $\alpha \lesssim \alpha_c$. The leading four-fermion operator will then have an effective scaling dimension $4+\eta$, $\eta \ll 1$. I will attempt to mimic the effect of the large anomalous dimension in diagrams by simply multiplying any loop containing an O_- insertion by $(k/\Lambda)^{\eta-2}$, where k is the loop momentum. Now, while O_- acquires a large negative anomalous dimension, O_+ acquires a large positive anomalous dimension. As a result, it will disappear from the theory even more quickly than in the weak-coupling limit of the walking gauge interactions, and we can ignore it. What about penguins? If we examine the diagrams of fig. 2, then the loop

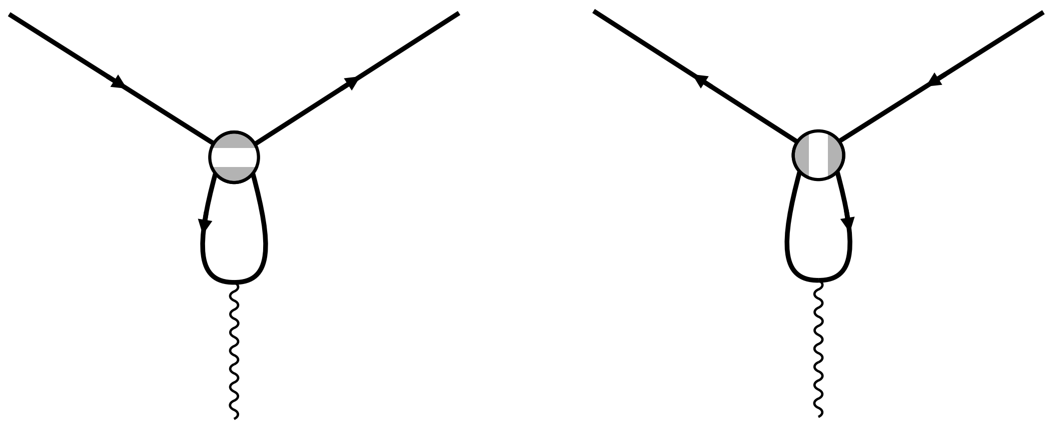


Figure 2 The penguin diagrams of the walking gauge interactions.

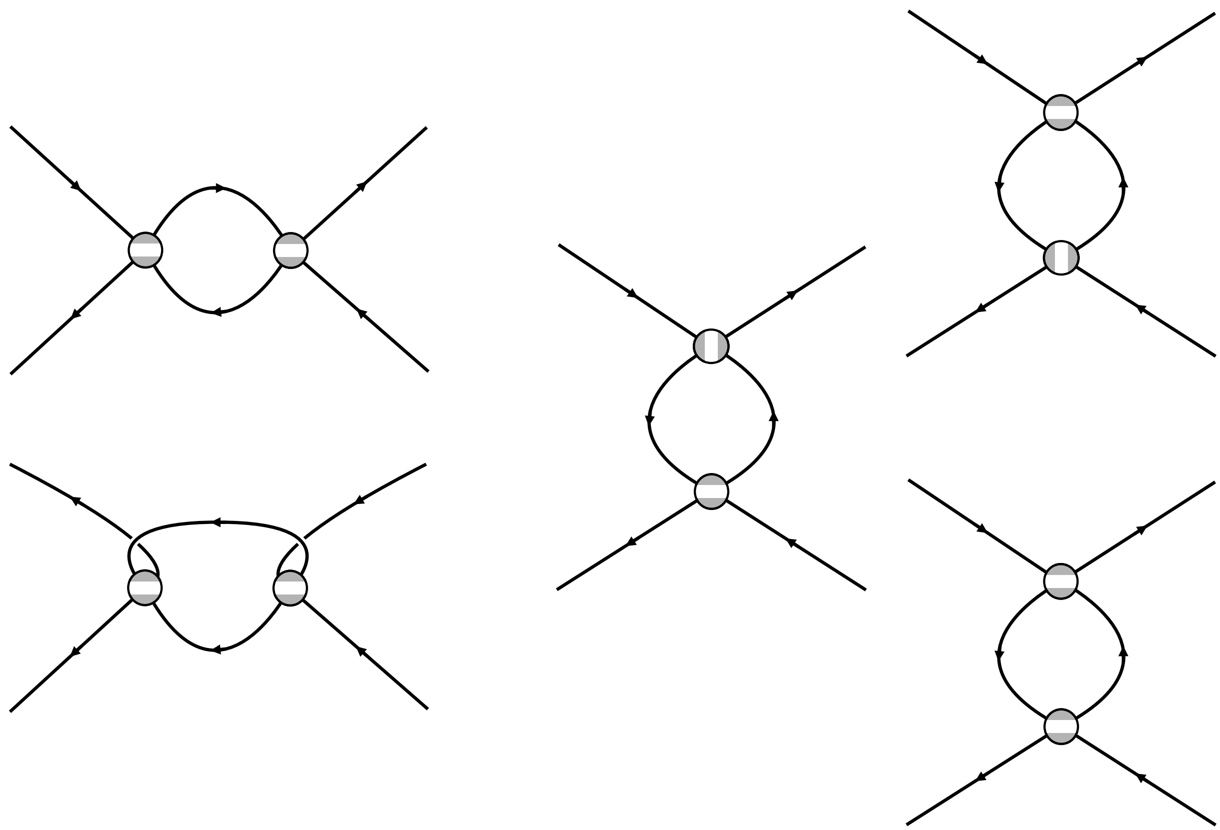
is naively quadratically divergent. Gauge invariance (in the form of derivatives in the penguin) reduces this to a logarithmic divergence in the weak-coupling case, so that the diagram indeed generates a mixing of the flavor-singlet piece of O_- with the WGI penguin. In the assumed strong-coupling scenario, however, there are nearly two additional powers of the loop momentum, so that the diagram becomes convergent, no mixing is generated, and we can safely ignore the penguin.

Note also that while certain operators with more fermion fields also may acquire negative large anomalous dimensions, their over-all scaling dimension will presumably remain larger than four, and hence it is consistent for them to be small. For example, the third power of the fermion mass operator, $(\overline{\Psi}\Psi)^3$, might well acquire an anomalous dimension of -3 as $\alpha \to \alpha_c$; but this will still leave it an operator of dimension six, and hence irrelevant.

For nearly marginal operators, there are in principle important logarithmic self-renormalization effects. To compute the beta function for the four-fermion coupling f_- associated with O_-, we must consider the graphs depicted in figs. 3. We can put in an explicit cutoff $p \ll \mu \ll \Lambda$, and differentiate with respect to $\ln \mu$; this leads to the following expressions,

$$\begin{aligned} \beta_- &= \frac{l+1}{2l}\left[3l + 7 - N_w(l-3)\right] \ , \\ \beta(f_-) &= \eta f_- - \frac{\beta_-}{32\pi^2} f_-^2 \ . \end{aligned} \tag{16}$$

(Given the treatment of the anomalous dimension here, the wavefunction renormalization graphs of fig. 4 have the wrong Lorentz structure to contribute, and vanish in the presence of a cutoff.) In computing the second term, I have set $\eta = 0$ since it leads only to terms of $\mathcal{O}(f_-^2 \eta)$, which is effectively higher order. I have also ignored any mixing induced by these diagrams with O_+, since the latter has a very large effective scaling dimension due to the walking gauge interactions. Since



(a) (b)

Figure 3 Contributions to the self-renormalization of the four-fermion operator.

the operator O_- started out only nearly marginal rather than exactly marginal, the beta function starts out at order f_- rather than f_-^2. The first term, as expected, has the same sign as the coupling itself, and so favors an infrared-free coupling. The second term, however, is strictly negative for appropriate choices of l and N_w; if $f > f_c = 32\pi^2\eta/\beta_-$ it will overcome the leading term, and the beta function will be negative. For an attractive coupling, $f > 0$, this implies asymptotic weakness, or more importantly for us, a large value in the infrared.

Within the rainbow approximation considered here, the value of the coupling f_- has no effect on the chiral-symmetry breaking scale; so if the WGI coupling walks slowly enough, f_- can grow to be quite large down at the scale Λ_χ where α finally exceeds α_c, and the chirally-symmetric vacuum becomes unstable. At this scale, the WGI binds the fermions into Goldstone bosons, and a spectrum of mesons. What is the effect of O_- on the masses of the vector mesons? Now, it is a composite operator, and it does not make sense to treat it as a product of currents; but if we fierz

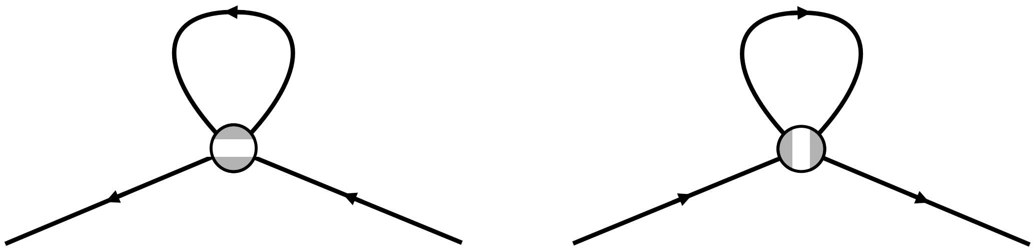


Figure 4 Contributions of the four-fermion operator to the fermion wavefunction renormalization.

the terms into products of WGI-singlets,

$$\frac{f_-(\Lambda_\chi)}{\Lambda_\chi^2}\left(\frac{\Lambda_\chi}{\Lambda}\right)^\eta \left[\frac{l+1}{l}\overline{\Psi}T^a\gamma_\mu P_L\Psi\,\overline{\Psi}T^a\gamma^\mu P_L\Psi - \frac{l^2-1}{2l^2}\overline{\Psi}\gamma_\mu P_L\Psi\,\overline{\Psi}\gamma^\mu P_L\Psi\right] , \tag{17}$$

and indulge in the treif temptation to split the operator anyway, then we will see by analogy with equation (1) that the attractive interaction in the $SU(l)$-adjoint channel gives the corresponding vector mesons a large negative contribution to their mass, while the repulsive interaction in the $SU(l)$-singlet channel presumably gives the singlet meson a positive contribution to its mass. The constituent mass of the Ψ plays the role of M in the original Lagrangian (1). Of course, the axial-vector state receives other contributions to its mass from physics associated with the spontaneous breaking of the axial symmetry; but the pure vector state should be unaffected, and thus light. The (global) vector symmetry remains exact ignoring weaker interactions, and can play the role of a custodial symmetry.

The Lagrangians of BHL-type models also respect chiral symmetry, but their four-fermion operators are precisely the $R\otimes\overline{R}$ (or equivalently left-right mixing) operators we sought to avoid earlier. If we add such operators to a walking gauge theory, it is plausible that their main effects will be in scalar rather than vector channels.

The models with vector-channel four-fermion operators have three different energy regimes: (i) the region stretching from the high-energy cut-off Λ, down to a scale Λ_s where the four-fermion operators become strong, in which all interactions other than the walking gauge interaction are weak and can be discussed sensibly in perturbation theory (in particular, explicit breaking of the chiral symmetries of the WGI can be discussed perturbatively), and in which some of the four-fermion operators have negative beta functions and thus grow as we scale down to lower energies; (ii) the region stretching from the scale Λ_s (where some of the four-fermion couplings become appreciable) down to a chiral symmetry-breaking or confinement scale Λ_χ, in which several interactions are strong and only symmetry arguments will really tell us anything; (iii) the region below Λ_χ, in which we will have a chiral effective Lagrangian with its derivative expansion, along with a light vector field with (weak) gauge-like couplings, which can again be treated in perturbation theory.

To introduce ordinary fermions into the theory, we may add terms of the following form to $\mathcal{L}$,

$$\frac{1}{\Lambda^2}\overline{\Psi}T^a\gamma_\mu P_L\Psi\,\overline{\psi}_L T^a\gamma_\mu\psi_L\ , \tag{18}$$

where ψ is an ordinary light fermion. These will eventually yield couplings of the composite vector meson to the light fermion. (One presumably must ensure that these couplings are non-anomalous, else the light fermion loops will give an additional contribution of order the chiral-symmetry breaking scale times powers of the coupling to the vector mass.)

An unusual feature of these models in the first, high-energy, region is the possibility of having terms in the Lagrangian which violate the $SU(l)$ symmetry *explicitly*, so long as they are sufficiently small (in which case they should not disturb the scenario explained earlier). Among such terms we can consider terms which will give rise to light fermion masses,

$$\overline{\Psi}\Psi\,\overline{\psi}M^l\psi\ . \tag{19}$$

Indeed, if in building extensions of the Standard Model we follow the CTSM philosophy [19] of assuming that the only flavor-violating quantities in the high-energy theory are proportional to the up- and down-quark mass matrices, then all FCNCs in these models will be suppressed by the usual GIM mechanism [20].

I now return to the issue of the r coefficient. In general, the coupling f_- at the original cutoff scale Λ will have some finite value. Even if the anomalous dimension generated by the WGI alone is not sufficiently large to make the four-fermion operator O_- nearly marginal, the combination of the WGI and the four-fermion coupling itself might suffice. One might explore this possibility by examining the combined Schwinger-Dyson equations, but I will not do so here.

There is another possibility which is more attractive. We may note that none of the above discussion depended in any essential way on the eventual breaking of the chiral symmetry; the latter only served to define the scale at which the fermions become became bound into light vectors. This suggests that the same mechanism should also work in the rather different class of chiral gauge theories.

Consider, for example, $SU(N)$ theories with left-handed Weyl fermion content of l conjugates of the symmetric tensor S^{xy} and $l(N+4)$ fundamentals Ψ_z. These theories contain an $SU(l)\times SU(l[N+4])\times U(1)$ global symmetry, with the symmetric tensors carrying a charge of -1 and the fundamentals a charge of $(N+2)/(N+4)$. In discussing these models, I shall make two assumptions. The first is that the global $U(1)$ is not spontaneously broken; this is in fact the case at at large N

[21]. The second is that there is no critical coupling per se, but only a cross-over in description of the theory from the fundamental fields to composite ones at a scale of order the confinement scale This seems reasonable in view of the fact that, given the first assumption, there is no Lorentz- and gauge-invariant order parameter distinguishing the high-energy from the low-energy theory. It may make sense to apply similar assumptions to some of the so-called moose models [22].

In this case, it appears sensible to define instead the 'critical coupling' α_c' as the value of the gauge coupling α at which the leading chirally-invariant four-fermion operator becomes exactly marginal. For general N, this operator is

$$O_a = \frac{1}{\Lambda^2}\overline{S}_{xy}T^a\gamma_\mu S_L^{x\hat{y}}\,\overline{\Psi}^y T^a\gamma^\mu\Psi_{L\hat{y}}\ , \tag{20}$$

where T^a are the generators of $SU(l)$ or one of its subgroups, possibly in a direct sum of copies of the fundamental representation when acting on the Ψ fields. In this theory, there simply is no fermion bilinear, and so we need not worry about the r coefficient. The beta function for this coupling is a bit different from that given in equation (16), as only the diagrams of fig. 3 (a) will contribute, and the analog of $\beta_- = (3l^2+8N^2+2+N(14+l^2))/(l(N+1))$. (This operator will mix with current-current type operators containing only symmetric tensors or only fundamental fermions; but as these have smaller WGI-generated anomalous dimensions in the rainbow approximation and are thus plausibly not nearly marginal, I will ignore them. It will also mix with an operator with the same fields and color structures as that of equation (20), but with the identity replacing the flavor matrices. This operator O_1 has the same WGI-generated anomalous dimension as O_a, and thus will be generated by renormalization-group flow. It turns out, however, that the relevant beta-coefficient matrix has only one positive eigenvalue; the corresponding infrared-attractive eigenvector is predominantly O_a, with a small repulsive admixture of O_1, which leaves the qualitative conclusions of equation (17) intact.)

Given the assumption that the $U(1)$ global symmetry is unbroken, there will necessarily be massless fermions in the theory. The most likely possibility is the straightforward generalization of the $l = 1$ model, in which the light fermions transform as an antisymmetric tensor under a surviving $SU(N + 4)$ symmetry. In the case considered here, we will presumably have l copies of such an antisymmetric tensor. If $l = 2$, it is possible, and in the presence of a strong four-fermion operator of the form in equation (20) even plausible, for an $SU(2)$ global symmetry to be preserved. (For larger l it does not seem possible for the $SU(l)$ to be preserved; presumably some maximal vector-like subgroup will be. This subgroup may well depend on the four-fermion operators present in the theory at the confinement scale.).

As an example of a walking chiral gauge theory, we can take a model of this class with $N = 4$, and $l = 2$. In this case, if we assume that the critical coupling α'_c is given by $\gamma_0 = -1$, we find

$$\alpha'_c = 0.47, \qquad \beta(\alpha'_c)/\alpha'_c = -0.15\ , \tag{21}$$

with the two-loop formula. (If we prefer to use the three-loop MS-scheme formula [23], we can take $N = 7$ and $l = 2$ as an example, for which $\alpha'_c = 0.27$ and $\beta(\alpha'_c)/\alpha'_c = -0.24$.)

The general outline of the discussion in vector-like models continues to apply, so long as we assume that a large value for the four-fermion coupling does not trigger 'premature confinement': then the value of the four-fermion coupling at the confinement scale where we trade the high-energy description for the low-energy one will be large, and correspondingly the vector meson will be light compared to the confinement scale.

In this particular $SU(4)$ model, we will also have 28 fermions transforming as doublets under the $SU(2)$ to which the light vector meson couples, and as an antisymmetric tensor under the suriving $SU(8)$ global symmetry. This is an attractive feature of these models: they naturally generate light fermions (massless in the absence of explicit breaking of the $U(1)$) which transform non-trivially under the effective gauge symmetry. For example, the left-handed fermions of the Standard Model might be composites, while the right-handed ones remain fundamental; mass terms could emerge from four-fermion terms in the high-energy theory which violate the $SU(2)$ symmetry explicitly. An amusing variant of Kaplan's mechanism [24] in technicolor theories, in which ordinary fermions acquire their masses through mixing with technifermions via higher-dimension operators, may be possible, in which it is not the different naive dimensions of these operators that leads to a hierarchy but rather the different WGI-induced anomalous dimensions.

More generally, we can also consider models which have both chiral and vector-like symmetries, for example:

$$\begin{aligned} SU(N)_w : &\qquad l \times S \oplus [l(N+4)+v] \times F \oplus v \times \overline{F}\ ; \\ \text{or} & \\ SU(N)_w : &\qquad l \times A \oplus [l(N-4)+v] \times F \oplus v \times \overline{F}\ . \end{aligned} \tag{22}$$

Here, S and A are the conjugates of the symmetric and antisymmetric tensor representations, respectively. In these models, there is again a chiral-symmetry breaking scale associated with the vector-like piece, and thus an r coefficient, but it can be near (or even larger than) 2; indeed, for models with a symmetric tensor,

$$r = \frac{2(N+2)}{N+1} \tag{23}$$

for the four-fermion operators of the kind given in equation (20), while for those with an antisymmetric tensor,

$$r = \frac{2(N-2)}{N-1} \tag{24}$$

for similar four-fermion operators with the symmetric tensor replaced by an antisymmetric one (and with $N > 6$; for $N = 5$ the leading operator involves only antisymmetric tensors while for $N = 6$ there are two leading operators, one of each kind, which presumably mix). In the large-N limit, $r = 2$ and both α_c and α_c' are $2\pi/(3N)$. Furthermore, if we take $l = 2$ and $v = (1+\epsilon)N + O(1)$, we find

$$\beta(\alpha_c)/\alpha_c = -\frac{11}{27} + \frac{25}{54}\epsilon \tag{25}$$

if we use the two-loop formula,

$$\beta(\alpha_c)/\alpha_c = -\frac{473}{2916} + \frac{463}{729}\epsilon - \frac{14}{729}\epsilon^2 \tag{26}$$

if we use the three-loop MS one. In addition to the possibility of explicit breaking of the eventual gauge-like interaction, one should also keep in mind that different members of an electroweak doublet do not have to have a common origin at high energy; for example, in such models with only partially-chiral representations, it may be possible for the top quark to be a WGI-baryon, while the bottom quark remains a light composite. (This would violate the custodial $SU(2)$, of course, but we already know that the top quark mass is large. Whether this is consistent with a light W depends on the dynamical competition between the attractive four-fermion interactions and the WGI.)

Finally, I will mention a more extreme yet richer scenario that may be possible in theories in which the running of the four-fermion coupling at large coupling is itself dependent on the matter content of the theory (as would be the case, for example, for a variety of moose models). For a suitable choice of matter representations, it too may be able to 'walk'. In such a case (assuming again that there is no 'premature confinement'), there is plausibly a critical value for the four-fermion coupling at which operators with yet larger numbers of fermion fields become marginal, and their dynamical evolution must be considered as well.

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